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WS 1999/00

Tutorial *Dynamical Systems: Theory and Numerics*

Return: 2pm, Monday, 8.11. metal box: Junk (48, ground floor)

Exercise 1:

- Which of the following functions are solutions of the differential equation $y'' - y = 0$? (a) e^x, (b) $\sin x$, (c) $4e^{-x}$, (d) 0, (e) $\frac{1}{2}x^2 + 1$.
- Which of the following functions are solutions of the differential equation $y'' - 4y' + 4y = e^x$? (a) e^x, (b) e^{2x}, (c) $e^{2x} + e^x$, (d) $e^{2x} + xe^x$.

Exercise 2:

Give a differential equation of the first order for the following families of curves (parameter $c \in \mathbb{R}$). Sketch the direction fields.
(a) $y = cx^2$, (b) $y = cx^2 + c$, (c) $y = cx^2 + \operatorname{sign}(c)c^2$.

Exercise 3:

Under certain conditions on f and g, the solution of $y' = f(x)g(y)$ with the condition $y(\xi) = \eta$ is given by

$$\int_\eta^y \frac{ds}{g(s)} = \int_\xi^x f(t)\, dt.$$

Using this method, which is called **separation of variables**, find all solutions of
(a) $y' + g(x)y = 0$, (b) $y' = y^2x^3$, (c) $y' = \frac{e^x}{y}$, $y(0) = 1$

Exercise 4

The differential equation $y' = f(y + ax + b)$ can be reduced to the case discussed in Exercise 3 with the substitution $u(x) = y(x) + ax + b$. Using this approach, solve the equation $y' = (x + y)^2$.

Exercise 5

The differential equation $y' = f(x/y)$ can also be reduced to the case discussed in Exercise 3 if we use the substitution $u(x) = y(x)/x$. Apply this approach to
(a) $y' = 2xy/(y^2 - x^2)$, (b) $y' = (x^2 + y^2)/xy$, $y(1) = -2$.