Dr. M. Junk WS 2001/02-9

Tutorial *Analysis I*

Return: 12:00, Friday 18.01.2002

Exercise 34:

Let $S \subset \mathbb{R}$ and $f : S \to \mathbb{R}$ be uniformly continuous. Show that:

(i) If (x_n) is a Cauchy sequence with $x_n \in S$ for all n then $(f(x_n))$ is also a Cauchy sequence.

(ii) If S is bounded then f is bounded.

Exercise 35:

Use the $\epsilon - \delta$ condition to show that the following functions are uniformly continuous:

$$f(x) = 3x + 11, \quad x \in \mathbb{R}, \qquad f(x) = \sqrt{x}, \quad x \in [0, \infty), \qquad f(x) = x^4, \quad x \in [0, 2].$$

Exercise 36:

Let $f : \mathbb{R} \to \mathbb{R}$ be uniformly continuous. Show that there exist constants $c, d \in \mathbb{R}$ such that $|f(x)| \leq c|x| + d$ for all $x \in \mathbb{R}$.

Exercise 37:

Let $f, g : S \to \mathbb{R}$ be uniformly continuous functions on $S \subset \mathbb{R}$. Show that $f + g$ is also uniformly continuous on S. Prove that corresponding statements are true for $f \cdot g$ and f/g – if necessary, you can use additional assumptions on f and g.

Exercise 38:

Show: f is continuous on $(a, b) \subset \mathrm{dom}(f)$ if and only if $\lim_{x \to \bar{x}} f(x) = f(\bar{x})$ for all $\bar{x} \in (a, b)$.