

Mathematik I

für die Studiengänge Chemie, Life Science und Nanoscience

Freiwillige Zusatzaufgaben zu Approximation von Funktionen

Lösungen

$$(1) \quad \sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}, \quad \cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

$$(2) \quad p_2(x, y) = y + \frac{\pi}{2} - x^2$$

$$(3) \quad \sum_{k=0}^{\infty} \frac{z^k}{k!} = \exp(z) = i \Rightarrow \mathbb{L} = \left\{ z = \frac{(4k+1)\pi}{2} i : k \in \mathbb{Z} \right\}$$

$$(4) \quad p_2(x, y) = y + (x-1)y + \frac{1}{2}y^2$$

$$(5) \quad p_2(x, y) = 2 + \frac{1}{2}(x-1) + \frac{1}{2}(y-1) - \frac{1}{16}(x-1)^2 - \frac{1}{8}(x-1)(y-1) + \frac{3}{16}(y-1)^2$$

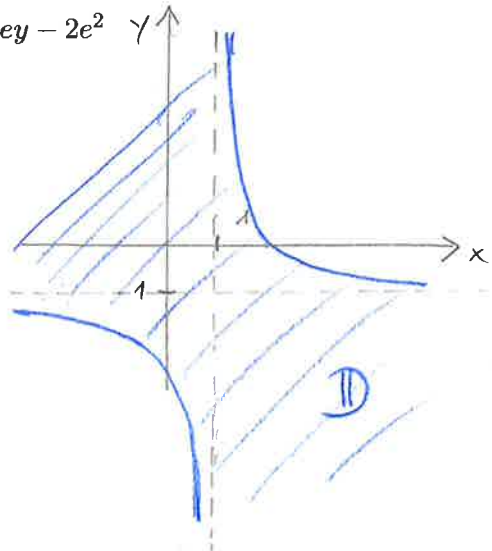
$$(6) \quad p_1(x, y) = e^2 + e^2(x-1) + 2e(y-e) = e^2x + 2ey - 2e^2$$

$$(7) \quad \text{a) } \mathbb{D} = \{(x, y) \in \mathbb{R}^2 : (x-1)(y+1) \leq 4\}$$
$$\mathbb{W} = [0, \infty)$$

b)

$$\nabla h(x, y) = \left(\frac{-y-1}{2\sqrt{4-(x-1)(y+1)}}, \frac{1-x}{2\sqrt{4-(x-1)(y+1)}} \right)$$

$$\text{Hess } h(x, y) = \begin{pmatrix} \frac{-(y+1)^2}{4\sqrt{4-(x-1)(y+1)}^3} & \frac{(x-1)(y+1)-8}{4\sqrt{4-(x-1)(y+1)}^3} \\ \frac{(x-1)(y+1)-8}{4\sqrt{4-(x-1)(y+1)}^3} & \frac{-(x-1)^2}{4\sqrt{4-(x-1)(y+1)}^3} \end{pmatrix}$$



$$(c) \quad p_2(x, y) = 1 - \frac{3}{2}(x-2) - \frac{1}{2}(y-2) - \frac{9}{8}(x-2)^2 - \frac{5}{4}(x-2)(y-2) - \frac{1}{8}(y-2)^2$$

$$(8) \quad p_2(u, v) = 2 + \frac{1}{2}(u-1) + \frac{1}{4}(v-2) - \frac{1}{16}(u-1)^2 - \frac{1}{16}(u-1)(v-2) - \frac{1}{64}(v-2)^2$$

(9)

$$\sum_{k=1}^{\infty} 2^k (x-1)^k = \frac{-2+2x}{3-2x}, \quad \text{Konvergenzbereich} = \left(\frac{1}{2}, \frac{3}{2}\right)$$

(10) **a)** $\mathbb{D} = \mathbb{R}^3 \setminus (0, 0, 2)$

b) $p_1(x, y, z) = \ln(2) + (x-1) - (z-1)$

c) $f(z) = \ln(4 + (z-2)^2)$

$$p_2(z) = \ln(8) - \frac{1}{2}z$$