Übungen zur **Mathematik I**für die Studiengänge **Chemie, Life Science und Nanoscience**Freiwillige Zusatzaufgaben zu **komplexe Zahlen****Lösungen**(1) a) Für  $z \in \mathbb{R}$ 

b)

$$z_1 = 3 \exp(0i) = 3$$

$$z_2 = 3 \exp\left(\frac{\pi}{2}i\right) = 3i$$

$$z_3 = 3 \exp(\pi i) = -3$$

$$z_4 = 3 \exp\left(\frac{3}{2}\pi i\right) = -3i$$

c)  $u_1 = 2, u_2 = -2, u_3 = \sqrt{3}i, u_4 = -\sqrt{3}i$ d)  $z = \frac{1+4k}{2} \pi i$  mit  $k \in \mathbb{Z}$ .(2)  $\exp(\pi i) = -1, \exp(2 + 2\pi i) = \exp(2), \ln(i) = \frac{\pi}{2}i, \ln(-2 + 2i) = \ln(2\sqrt{2}) + \frac{3}{4}\pi i,$ 

$$i^i = \exp\left(-\frac{\pi}{2}\right), \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^{2+i} = \exp\left(-\frac{\pi}{4}\right) \cdot i.$$

(3)  $z_1 = 1 + i, z_2 = 1 - i, z_3 = \frac{1}{2}, z_4 = -\frac{1}{2}.$ (4)  $z_1 = 3 + 2i, z_2 = -5i, z_3 = 3 - 2i, z_4 = 5i, z_5 = 0.$ 

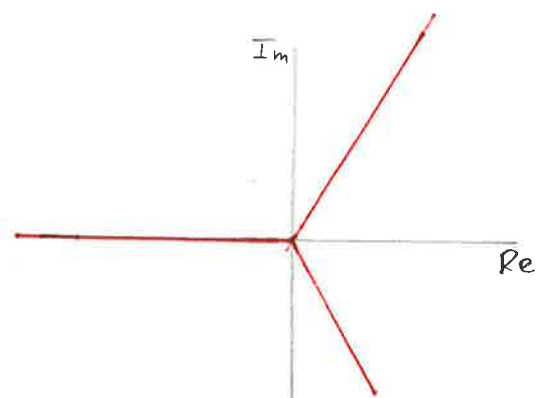
(5) (i)

$$z_1 = \sqrt{2} \exp\left(\frac{\pi}{4}i\right) = 1 + i$$

$$z_2 = \sqrt{2} \exp\left(\frac{3\pi}{4}i\right) = -1 + i$$

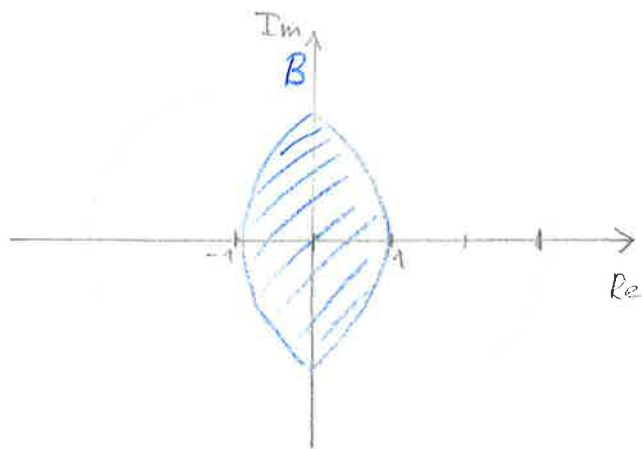
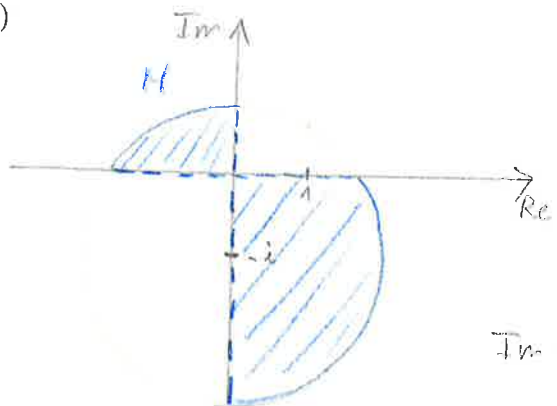
$$z_3 = \sqrt{2} \exp\left(\frac{5\pi}{4}i\right) = -1 - i$$

$$z_4 = \sqrt{2} \exp\left(\frac{7\pi}{4}i\right) = 1 - i$$

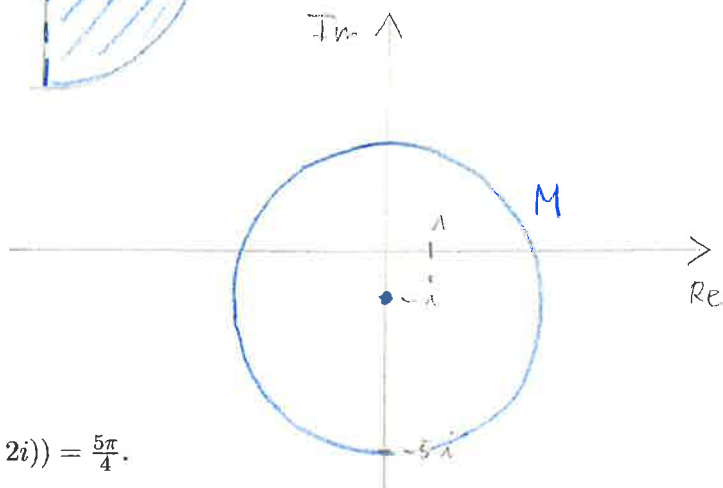
(ii)  $z = r \exp\left(\frac{(1+2k)\pi}{3}i\right)$  mit  $r \geq 0, k = 0, 1, 2.$ (6) (i)  $|u| = 1$ (ii)  $\operatorname{Im}(w) = -\frac{1}{32}$ 

bitte wenden

(7)



(8) a)



b)  $\text{Im}(\ln(-2 - 2i)) = \frac{5\pi}{4}$ .

c)

$$z_1 = 2 \exp\left(\frac{\pi}{6}i\right) = \sqrt{3} + i$$

$$z_2 = 2 \exp\left(\frac{5\pi}{6}i\right) = -\sqrt{3} + i$$

$$z_3 = 2 \exp\left(\frac{3\pi}{2}i\right) = -2i$$