University of Konstanz
Department Mathematics and Statistics
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Winter Semester 2017/2018

# Exercises for
# Theory and Numerics of Partial Differential Equations

http://www.math.uni-konstanz.de/numerik/personen/mechelli/teaching.html

## Exercise Sheet 10

**Submission[1]: 25$^{\text{th}}$ January at 10:00**

**Exercise 1.** (Theory, 10 points)
Consider the following problem:

$$\begin{cases} -\lambda\Delta u + cu = f & \text{in } \Omega \\ \lambda \frac{\partial u}{\partial n} + \alpha u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

with $\Omega \subset \mathbb{R}^d$ bounded, $f \in L^2(\Omega)$ , $\alpha \geq 0$, and $\lambda, c > 0$. Prove that:

1. Exists a unique weak solution $u \in H^2(\Omega)$ of (1).
2. Given $\{\Phi_1, \ldots, \Phi_N\}$ a base of $X_h \subset H^1(\Omega)$, the matrix of the equivalent linear equations' system is symmetric and positive definite.

( **Hint**: For 1. consider the Lax-Milgram Theorem. )

**Exercise 2.** (Programming, 10 points)
Consider the following problem:

$$\begin{cases} -\lambda\Delta u + cu = f & \text{in } \Omega = [0,1]\times[0,1] \\ \frac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega \end{cases} \tag{2}$$

with $f \in L^2(\Omega)$ and $\lambda, c > 0$. In `main_10_2`:

1. Generate a grid[2] $[x_0, \ldots, x_N] \times [y_0, \ldots, y_M]$ with $N, M \in \mathbb{N}$,
2. Solve (2) numerically with finite differences for the following $f$:
   (a) $f_1(x, y) = 1$
   (b) $f_2(x, y) = -6y^2(2y-3)(2x-1) - 6x^2(2x-3)(2y-1) + x^2(2x-3)y^2(2y-3)$
   (c) $f_3(x, y) = (8\pi^2 + 1)\cos(2\pi x)\cos(2\pi y)$

   for all $(x, y) \in \Omega$,
3. Plot the solutions.

[1] The Theory Exercises will be collected at the begin of the lecture. The Programming Exercises have to be sent by email to `luca.mechelli@uni-konstanz.de` (group of Tuesday) and to `hai.nguyen-pham@uni-konstanz.de` (group of Wednesday) **before** the submission's deadline.

[2] Consider the fact that the discretization step $dx$ may be different from $dy$.