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ÜBUNGEN ZUR VORLESUNG POSITIVE POLYNOME

BLATT 09

These exercises will be collected Tuesday 6th July in the mailbox n.14 of the Mathematics department.

Let V be a $\mathbb{R}$-vector space, $\dim V = \aleph_0 = |\mathbb{N}|$.

1. Let $W \subset V$ be a finite-dimensional subspace, $\dim W = n$.
Fix $\mathcal{B} = \{w_1, \dots, w_n\}$ a basis of W and define

$$\begin{array}{rcl} \Phi_{\mathcal{B}} \colon W & \longrightarrow & \mathbb{R}^n \\ \sum_{i=1}^{n} r_i w_i & \mapsto & (r_1, \dots, r_n) \end{array}$$

Show that:

(a) Setting

$(*)$ $$\mathcal{U} \subseteq W \text{ is open} \overset{\text{def}}{\iff} \mathcal{U} = \Phi_{\mathcal{B}}^{-1}(A), \text{ where } A \subseteq \mathbb{R}^n \text{ is open},$$

$(*)$ defines a topology τ on W.

(b) τ does not depend on $\mathcal{B}$ (τ is called **the Euclidean topology** on W).

(c) τ is a local convex topology.

(d) For every finite-dimensional subspaces $W_1 \subset W_2 \subset V$, the Euclidean topology on W_1 coincides with the subspace topology induced from the Euclidean topology on W_2.

2. Define

$$\mathcal{U} \subseteq V \text{ is open} \overset{\text{def}}{\iff} \begin{cases} \forall\, W \subset V \text{ finite-dimensional subspace, } \mathcal{U} \cap W \text{ is open} \\ \text{with respect to the Euclidean topology on } W. \end{cases}$$

(a) Show that this defines a topology τ (called **the finite topology** on V).

(b) Let $\{v_1, v_2, \dots,\}$ be a basis of V and set $V_n := \text{span}\{v_1, \dots, v_n\}$ (the subspace generated by $v_1, \dots, v_n$).
Show that $\mathcal{U} \subseteq V$ is open with respect to τ if and only if $\mathcal{U} \cap V_n$ is open in V_n with respect to the Euclidean topology on $V_n, \forall\, n \in \mathbb{N}$.

Definition. Let K be a topological field and V a K-vector space. We say that V is a **topological K-vector space** if there is a topology on V such that the maps

$$\begin{array}{ccc} V \times V & \longrightarrow & V \\ (v_1, v_2) & \mapsto & v_1 + v_2 \end{array}$$

$$\begin{array}{ccc} K \times V & \longrightarrow & V \\ (\lambda, v) & \mapsto & \lambda v \end{array}$$

are continuous.

3. Let W, W' finite-dimensional $\mathbb{R}$-vector spaces with the Euclidean topology. Show that:

(i) W is a topological $\mathbb{R}$-vector space.

(ii) $L\colon W \to \mathbb{R}$ linear $\Rightarrow$ L continuous.

(iii) $L\colon W \times W \to W'$ bilinear $\Rightarrow$ L continuous.