

# Sofia Kovalevskaya.

Mathematician, Writer,  
Revolutionary.

A decorative graphic element consisting of a solid teal horizontal bar, followed by three thin white horizontal lines, and a final teal horizontal bar, all extending from the left edge of the text area towards the right.

# Women in Maths Seminar 19/01/2016

- **Introduction.**
- **Her life.**
- **Her mathematical work.**
- **To be continued...**

# Introduction.

- A female mathematician.

A female mathematician.



# Introduction.

- A female mathematician.
- The Cauchy-Kovalevskaya Theorem.

# The Cauchy-Kovalevskaya Theorem.

Cas quasi-lin :  $Lu = \sum_{|K|=m} A_K D^K u + C = 0$   
 On suppose surf init  $S$  est anal réelle de 1 var d'1 descripteur  $x^0$   
 Sur  $S$ , données de Cauchy compact.  $D^\beta u$  ( $|\beta| < m$ ) qui est anal réelle en  $x^0$ .  $A_K, C$  sont anal réelles de leurs var.  $x$ ,  $D^\alpha u$  au pt  $x^0$ ,  $D^\beta u$   
 De plus,  $S$  caract. en  $x^0$  c-à-d  $Q(D^p) \neq 0$  où  $Q(x) = \sum_{|K|=m} A_K x^K$   
 et  $Q(S) = \det \sum_{|K|=m} A_K S^K$ .  
 Td'abord, on change  $x^0$  en 0 et  $S$  localement en axe transv.  
 anal d'un vois. de 0 de la plan  $x_n = 0$ . En remplaçant les der.  
 d'ordre  $\leq m$  par des nouvelles var dépendantes, on se ramène à syst d'éq  
 → forme standard  $\frac{\partial u}{\partial x_n} = \sum_{i=1}^{n-1} a_i(x, u) \frac{\partial u}{\partial x_i} + b(x, u)$   
 où  $a_i(x, u)$  met en jeu  $b(x, u)$  var. connue  
 Faut poser  $u = f$  comme nouvelle inconnue, on peut supposer  $f = 0$   
 On ajoute  $x_n$  comme n-ème var. dép.  $u^*$  au comp. de la  $u$  et  $\frac{\partial u^*}{\partial x_n} = 1$   
 et  $u^* = 0$  c-à-d.  $\Rightarrow a_i$  et  $b$  indep. de  $x_n$

Cauchy  $+++$  th =  $a_i$  et  $b_j$  sont anal réelles de  $(x_1, \dots, x_{n-1}, u_1, \dots, u_{n-1})$  en  $\mathbb{R}^{n-1}$   
 Kovalevskaya de  $\mathbb{R}^{n-1}$  Alors le syst  $\frac{\partial u_i}{\partial x_n} = \sum_{i=1}^{n-1} \sum_{|K| \leq m-1} a_i^K(x) \frac{\partial u^K}{\partial x_i} + b_j(x)$  j=1...n

avec cond. init  $u_j = 0$  par  $x_n = 0$   $j=1, \dots, n$  à un syst  
 unique de sol  $u_j(x_1, \dots, x_n)$  qui est anal. en  $\mathbb{R}^n$ .  
 (preuve:  $\exists$  sol  $u_j(x)$  anal telle  $\forall x \in \mathbb{R}^n \Rightarrow$   
 (a)  $D_\alpha D^\beta u_j(x) = P_\alpha(D^\beta a_i^K(x), D^\beta b_j(x), D^\beta u_j(x))$  où  
 $P_\alpha = (\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_{n-1}})$  (grad)  $\beta \in \mathbb{Z}^{n-1}$   $|\beta| \leq |\alpha| \leq |\beta| + 1$   $\beta \in \mathbb{Z}^n$   
 $P_\alpha$  poly à coeff  $\in \mathbb{N}$ .  
 De plus  $D^\alpha u_j(x) = 0$  par  $x_n = 0 \Rightarrow D^\alpha u_j(x)$  est fait de  $a_i^K(x), b_j(x)$   
 $\Rightarrow u_j(x)$  est de var. rég. Répét. si on calcule  $u_j(x)$   
 à partir de (a) ( $C_j^0 = D^\alpha u_j(x)$ ) et si  $\sum_{i=1}^{n-1} \frac{1}{2^i} C_j^i x^i$  c-à-d.  
 et veut  $u_j(x)$  un vois. de 0 dans  $\mathbb{R}^n$  tel que  $u_j(x) = 0$   
 donc unq série puis formelle par  $u_j(x)$  de coeff  $\frac{D^\alpha u_j(x)}{|\alpha|!}$  conv  
 au vois. de 0  $\Rightarrow$  méth des maj  
 Soit  $a_i^K(x) = A_i^K(x) b_j^K(x) \ll B_j^K(x)$  et  $U_j(x)$  sol  $\frac{\partial U_j}{\partial x_n} = \sum_{i=1}^{n-1} \frac{A_i^K(x)}{B_j^K(x)} \frac{\partial U_j}{\partial x_i} + B_j^K(x)$   $j=1, \dots, n$   
 $U_j = 0$  par  $x_n = 0$   $j=1, \dots, n$   
 Alors  $|D^\alpha u_j(x)| \ll |D^\alpha U_j(x)|$  et tout OK  
 Supposons  $a_i^K(x) \text{ et } b_j^K(x) \in C^{M, N}(0)$  et on majore  $M, N$   
 $\Rightarrow \frac{\partial U_j}{\partial x_n} = \frac{M, N}{2^{-x_1} \dots 2^{-x_{n-1}} - U_1 - \dots - U_{n-1}} (1 + \sum_{i=1}^{n-1} \sum_{|K| \leq m} \frac{\partial U_j}{\partial x_i})$  est prod  
 $U_j = 0$  par  $x_n = 0$   $j=1, \dots, n$  de Cauchy majoré  
 qui a pour sol  $U_j(x_1, \dots, x_n) = V_j(x_1, \dots, x_{n-1}, x_n)$   $j=1, \dots, n$   
 où  $V_j(s, t)$  est prod de Cauchy  $V_j = \frac{M, N}{2^{-s} - N V}$   $(1 + N(n-1)V)$   
 $\Rightarrow$  sol  $V_j(s, t) = \frac{1}{N} (n-s - \sqrt{(n-s)^2 - 2nMNt})$  est anal réelle en  $(s, t)$   
 $V(s, t)$  dépend de  $M, N \Rightarrow V(s, t) \in C_{p, q}$  (à  $U_j$  résolvant de  $M, N, n$ )  
 Le théo en puis de  $V_j(s, t)$  conv pour  $|s| + |t| < \rho$   
 où  $\rho$  dépend des  $N, n-1$  var et de  $C^{M, N}(0)$  à l'équale  $\exists \frac{1}{2^i}$  et  $b_j^K(x)$   
 Formule de Green-Lagrange  
 th de dir de Gauss  $\int_{\partial \Omega} D_\alpha u(x) dx = \int_{\partial \Omega} u(x) \frac{\partial x_i}{\partial n} dx = \int_{\partial \Omega} u(x) S_i dx$   
 où  $\frac{1}{\partial n}$  dir. de la dir normal  $S = (S_1, \dots, S_n)$  de  $\partial \Omega$   $dx$  élem de surf en  $x$   
 ( $\partial \Omega$  assez rég. pour que le th. s'applique à  $u \in C^1(\bar{\Omega})$ )

# Introduction.

- A female mathematician.
- The Cauchy-Kovalevskaya Theorem.
- A socialist militant.

# A feminist and socialist militant.

Louise Michel



Alexandra  
Kollontai



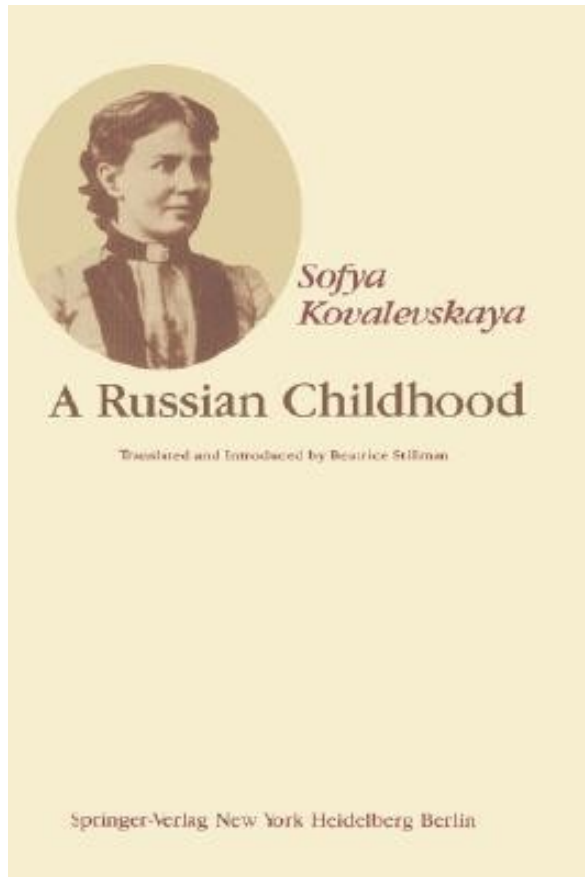
Clara Zetkin



# Introduction.

- A female mathematician.
- The Cauchy-Kovalevskaya Theorem.
- A socialist militant.
- An intellectual novelist.

An intellectual novelist.



# Introduction.

- A female mathematician.
- The Cauchy-Kovalevskaya Theorem.
- A socialist militant.
- An intellectual novelist.
- A great mathematician.


# A great mathematician.



http://www-groups.dcs.st-and.ac.uk/~7/history/Mathematicians/K

## Sofia Vasilyevna Kovalevskaya

Born: 15 Jan 1850 in Moscow, Russia  
Died: 10 Feb 1891 in Stockholm, Sweden



Click the picture above  
to see six larger pictures

[Show birthplace location](#)

[Previous](#) (Chronologically) [Next](#) [Biographies Index](#)

[Previous](#) (Alphabetically) [Next](#) [Main index](#)

Sofia Kovalevskaya was the middle child of Vasily Korvin-Krukovsky, an artillery general, and Yelizaveta Shubert, both well-educated members of the Russian nobility. Sofia was educated by tutors and governesses, lived first at Palabino, the Krukovsky country estate, then in St. Petersburg, and joined her family's social circle which included the author Dostoevsky.

Sofia was attracted to mathematics at a very young age. Her uncle Pyotr Vasilievich Krukovsky, who had a great respect for mathematics, spoke about the subject. Sofia wrote in her autobiography:

*The meaning of these concepts I naturally could not yet grasp, but they acted on my imagination, instilling in me a reverence for mathematics as an exalted and mysterious science which opens up to its initiates a new world of wonders, inaccessible to ordinary mortals.*

When Sofia was 11 years old, the walls of her nursery were papered with pages of Ostrogradski's lecture notes on differential and integral analysis. She noticed that certain things on the sheets she had heard mentioned by her uncle. Studying the wallpaper was Sofia's introduction to calculus.

It was under the family's tutor, Y. I. Malevich, that Sofia undertook her first proper study of mathematics, and she says that it was as his pupil that

*I began to feel an attraction for my mathematics so intense that I started to neglect my other studies.*

Sofia's father decided to put a stop to her mathematics lessons but she borrowed a copy of Bourdeu's *Algebra* which she read at night when the rest of the household was asleep.


A year later a neighbour, Professor Tyrtov, presented her family with a physics textbook which he had written, and Sofia attempted to read it. She did not understand the trigonometric formulae and

1 of 4

# 1) Her life.

- Sof'ya Vasil'evna Kovalevskaya (1850 -1891)





**1850-1868**  
**A Russian childhood.**

## A Russian childhood: minor gentry.

Elizaveta Schubert



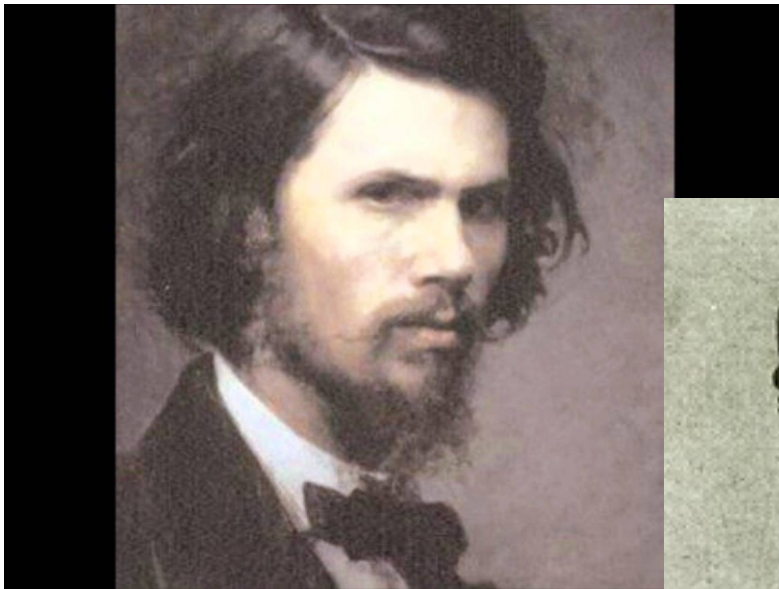
Vasily Korvin-Krukovsky



Aniouta

+ her younger brother: Fyodor

## A Russian childhood: intelligentsia.



Dostoevsky



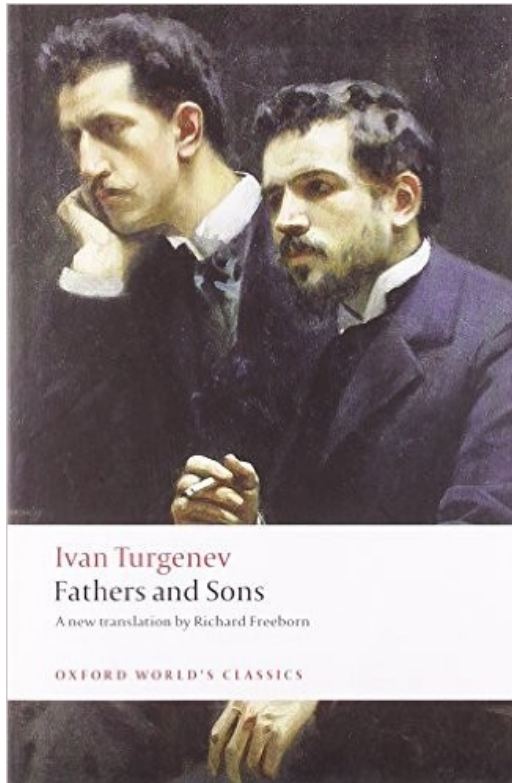
Julia Lermontova



Anna Filosofova



## A Russian childhood: nihilism.



WHAT'S TO BE DONE?

A ROMANCE.

N. G. TCHERNYSHEVSKY.

TRANSLATED BY  
HENRY E. TUCKER.

FOURTH EDITION.

NEW YORK:  
HARVARD BOOK CO.,  
1891.

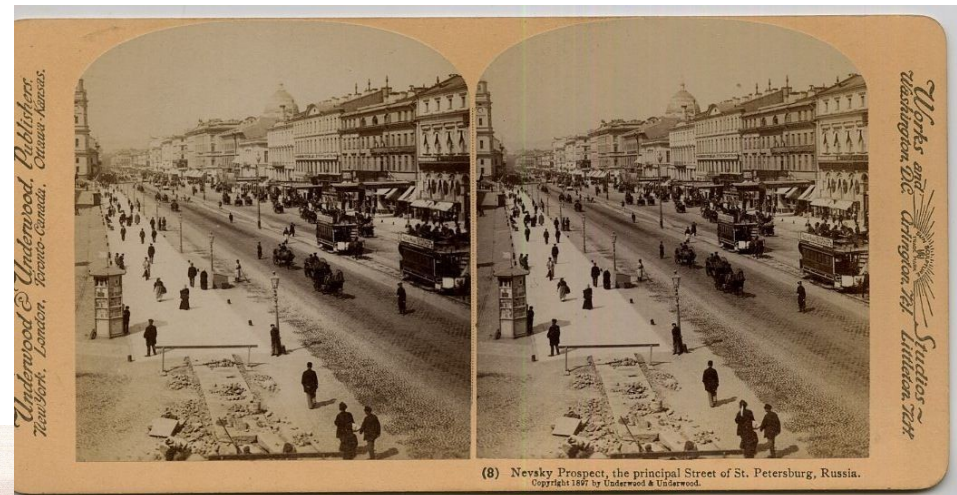
Mariage blanc with a young nihilist  
scientist in Sept. 1868:

**Vladimir Kovalevsky**

1869-1874  
A European student.

A European student.

- St Petersburg  
(fall 1868)  
-> met Chebyshev



Vienna  
(early 1869)  
-> too expensive

A European student.

- Heidelberg (fall 1869 – fall 1870)
  - Physics: Bunsen – Kirchhoff – Helmholtz
  - Maths: du Bois-Reymond – Königsberger



A European student.

- London (Oct. 1869)
  - G. Eliot – Darwin – T. Huxley – H. Spencer



George Eliot



A European student.

- Berlin (1870 – 1874)



Weierstrass

## La Commune de Paris 1871

- join there Aniouta on 5th of April:
  - Sofia: nurse. Vladimir: biologist.
  - Aniouta: member of the Commune



## A Weierstrass student.

Winter 1870–1871	Elliptic Functions
Summer 1871	Recent Synthetic Geometry Selected Problems of Geometry and Mechanics Solvable Using Elliptic Functions
Winter 1871–1872	Abelian Functions
Summer 1872	Introduction to Analytic Functions Calculus of Variations
Winter 1872–1873	Elliptic Functions
Summer 1873	Elements of Recent Synthetic Geometry Selected Problems of Geometry and Mechanics Solvable Using Elliptic Functions
Winter 1874	Abelian Functions




## A Weierstrass Doctoral student.

- In 18 months,  
3 different  
works:

KOVALEVSKAYA, S. V.

1875	Zur Theorie der partiellen Differentialgleichungen. <i>JFM</i> 80; 1–32. Russian translation in <i>Raboty</i> , 7–50.
1884	Über die Reduction einer bestimmten Klasse abel'scher Integrale dritten Ranges auf elliptische Integrale. <i>AM</i> 4; 393–414. Russian translation in <i>Raboty</i> , 51–74.
1885a	Über die Brechung des Lichtes in crystallinischen Mitteln. <i>AM</i> 6; 249–304. Russian translation in <i>Raboty</i> , 75–138. French résumé in <i>CR</i> 98; 356–357. Swedish résumé in <i>Öfversigt af Kungl. Vetenskaps-Akademiens Förhandlingar</i> 41; 119–121.
1885b	Zusätze und Bemerkungen zu Laplace's Untersuchung über die Gestalt der Saturnringe. <i>Astronomische Nachrichten</i> 111; 37–48. Russian translation in <i>Raboty</i> , 139–152.
1886	Reminiscences of George Eliot (Russian). <i>Russkaya Mysl</i> 6; 93–108.

29/08/1974. Doctorate degree  
in absentia and « Summa cum Laude ».



1875-1883  
Hard times.

Back to Russia, family affairs.

- 1874: death of her father.
- No position in Russia
  - > real-estate investments with Vladimir
  - > **bankrupt !**
- Love with Vladimir.

Back to Russia, family affairs.

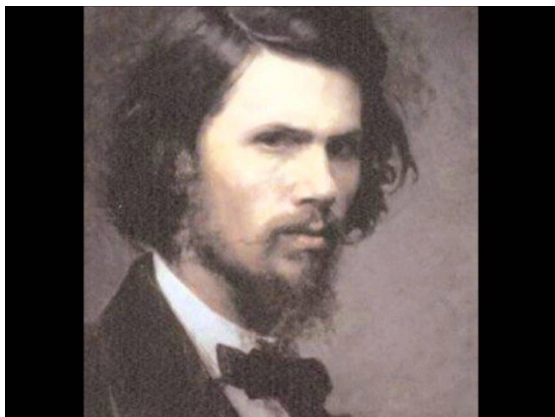
- 1878: birth of their daughter with Vladimir:  
**Sof'ya (Fufa)**
- 1879: death of her mother.



Back to Russia, family affairs.

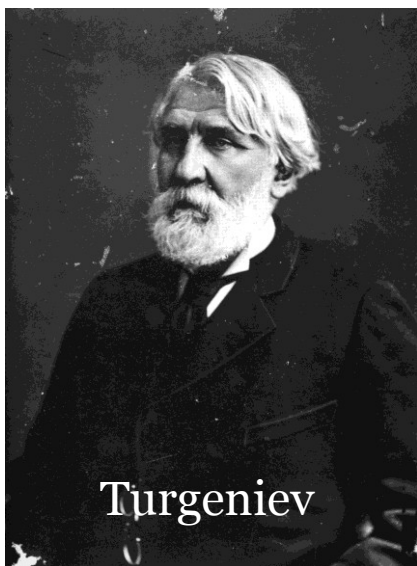
- 1881: Sofia leaves Vladimir and goes to Berlin, then Paris, with Fufa.
- 1883: new bankruptcy and prosecution for Vladimir, which commits **suicide** in April.

## From nihilism to socialism.



Yulia  
Lermontova

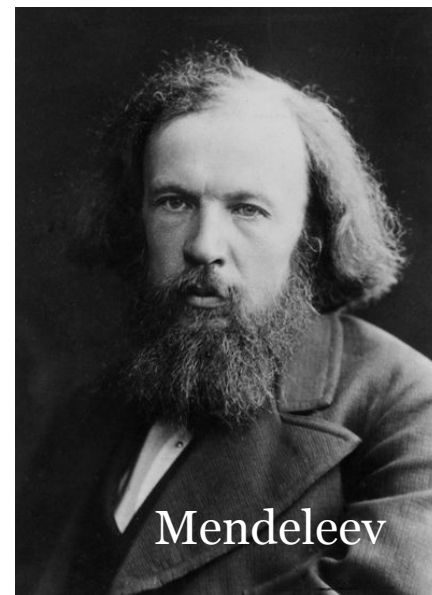
## Science and literary criticism



Turgeniev



Nekrassov



Mendeleev

## From nihilism to socialism.

- From « Going to the people » (1860's and 1870's) of *Land and Freedom*...



From nihilism to socialism.

- ... to the assassination of the czar Alexander II by the *Will of the People*.



Netchaiev



Vera Figner



# From nihilism to socialism.

Maria Jankowska-Mendelson



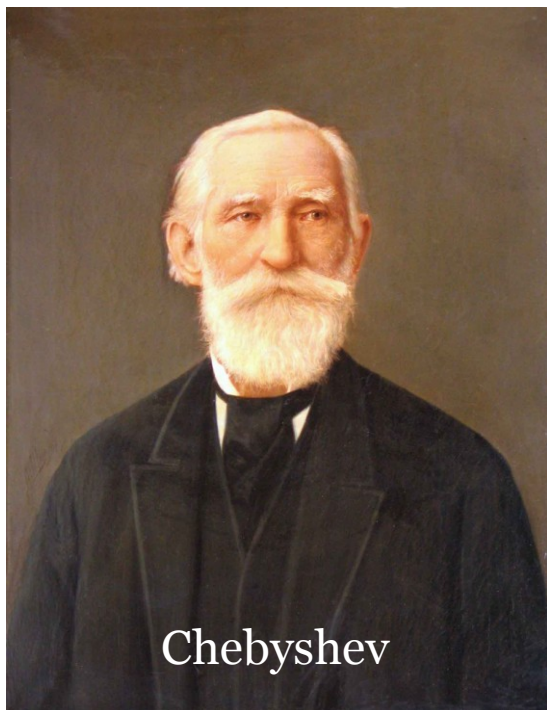
Georg Von Wollmar



Piotr Lavrov

## Mathematicians and mathematics.

- Return to Maths in 1879, on Lamé equations.



## Mathematicians and mathematics.

- 1976: St Petersburg
- 1979: St Petersburg
- 1881: Berlin, then Paris
- 1993: Berlin, then  
Odessa, then  
**Stockholm !**



Mittag-Leffler,  
her elder Brother

1884-1891  
A Professor.

## As a Mathematics Professor.

- Lectures at the Stockholm University.
- Editor of « Acta Mathematica ».
- A prominent researcher.

## As a Mathematics Professor.

KOVALEVSKAYA, S. V.

- 1875 Zur Theorie der partiellen Differentialgleichungen. *JFM* 80; 1–32. Russian translation in *Raboty*, 7–50.
- 1884 Über die Reduction einer bestimmten Klasse abel'scher Integrale dritten Ranges auf elliptische Integrale. *AM* 4; 393–414. Russian translation in *Raboty*, 51–74.
- 1885a Über die Brechung des Lichtes in crystallinischen Mitteln. *AM* 6; 249–304. Russian translation in *Raboty*, 75–138. French résumé in *CR* 98; 356–357. Swedish résumé in *Öfversigt af Kungl. Vetenskaps-Akademiens Förhandlingar* 41; 119–121.
- 1885b Zusätze und Bemerkungen zu Laplace's Untersuchung über die Gestalt der Saturnringe. *Astronomische Nachrichten* 111; 37–48. Russian translation in *Raboty*, 139–152.
- 1886 Reminiscences of George Eliot (Russian). *Russkaya Mysl* 6; 93–108.

## As a Mathematics Professor.

1889 Mémoire sur un cas particulier de la rotation d'un corps solide autour d'un point fixe. *AM* 12; 177–232. Russian translation of paragraphs 1–4 and 9 in *Raboty*, 235–244. The rest of the article is identical to 1890a.

1890a Mémoire sur un cas particulier du problème de la rotation d'un corps solide autour d'un point fixe, où l'intégration s'effectue à l'aide de fonctions ultraélliptiques du temps. *Mémoires Présentés par Divers Savants* 31; 1–62. Russian translation in *Raboty*, 153–220.

1890b Sur une propriété du système d'équations différentielles qui définit la rotation d'un corps solide autour d'un point fixe. *AM* 14; 81–93. Russian translation in *Raboty*, 221–234.

1890c An autobiographical sketch (Russian). *Russkaya Starina* 11; 450–463. English translation in *Stillman* 1978, 213–229.

1890d *Memories of Childhood* (Russian). *Vestnik Evropy* 7; 55–98, 8; 584–640. English translation in *Leffler* 1895 and in *Stillman* 1978.

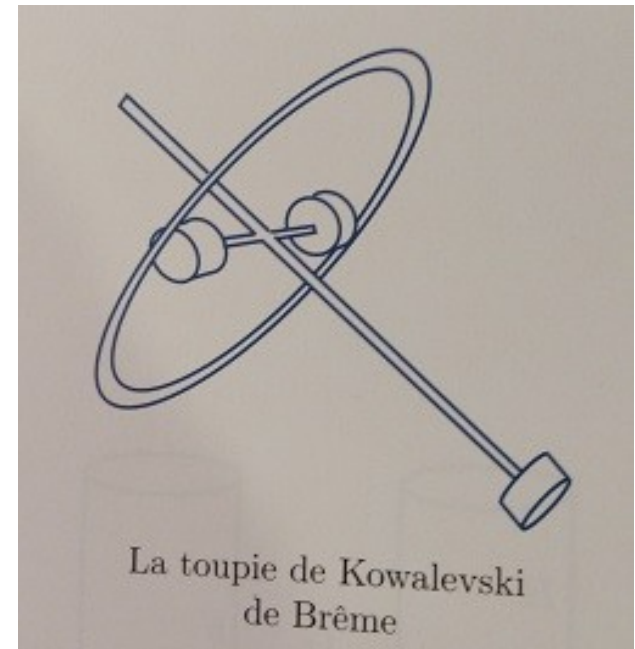
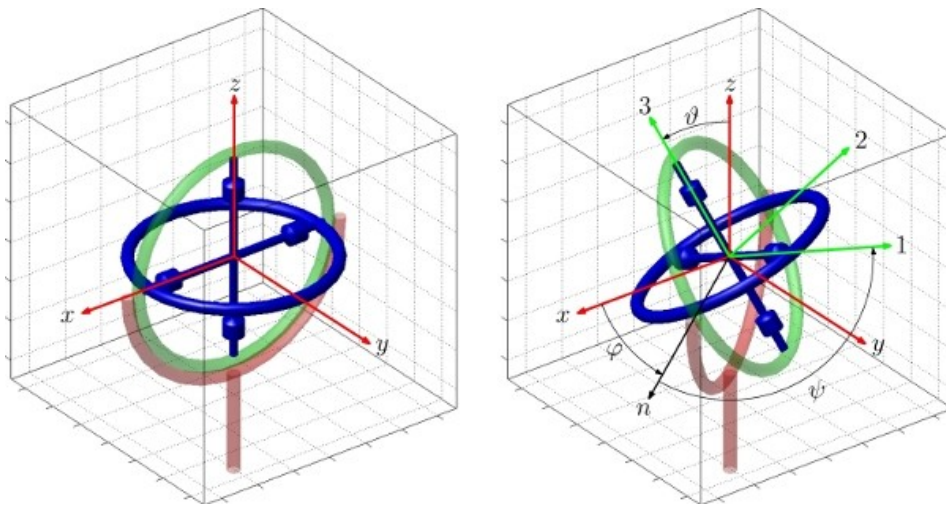
1891 Sur un théorème de M. Bruns. *AM* 15; 45–52. Russian translation in *Raboty*, 245–254.

1892 *The Nihilist Woman* (Russian). Geneva: Vol'naya Russkaya Tipografia.

Raboty *S. V. Kovalevskaya. Scientific Works* (Russian). Moscow: USSR Academy of Sciences, 1948.

Nachlass References to Kovalevskaya's unpublished material in the archives of the USSR Academy of Sciences, 1948, can be found in *Kochina* 1981 and *Koblitz* 1983. The material in the Institut Mittag-Leffler was described in *Grattan-Guinness* 1971.

# The Kovalevskaya Top.



« Die mathematische Nixe »



# 1888 The Bordin Prize.

SUR LE PROBLÈME DE LA ROTATION  
D'UN CORPS SOLIDE AUTOUR D'UN POINT FIXE<sup>1</sup>

PAR

SOPHIE KOWALEVSKI  
à STOCKHOLM.

§ 1.

Le problème de la rotation d'un corps solide pesant autour d'un point fixe peut se ramener, comme on sait, à l'intégration du système d'équations différentielles suivant:

$$(1) \quad \begin{aligned} A \frac{dp}{dt} &= (B - C)qr + Mg(y_0 r'' - z_0 r'), & \frac{dy}{dt} &= r' - q r'', \\ B \frac{dq}{dt} &= (C - A)rp + Mg(z_0 r'' - x_0 r'), & \frac{dz}{dt} &= p r'' - r r', \\ C \frac{dr}{dt} &= (A - B)pq + Mg(x_0 r'' - y_0 r'), & \frac{dx}{dt} &= q r'' - p r'. \end{aligned}$$

Les constantes  $A, B, C, Mg, x_0, y_0, z_0$ , qui figurent dans ces équations ont la signification suivante.

$A, B, C$  sont les axes principaux de l'ellipsoïde d'inertie du corps considéré, relativement au point fixe.

$M$  est la masse du corps;

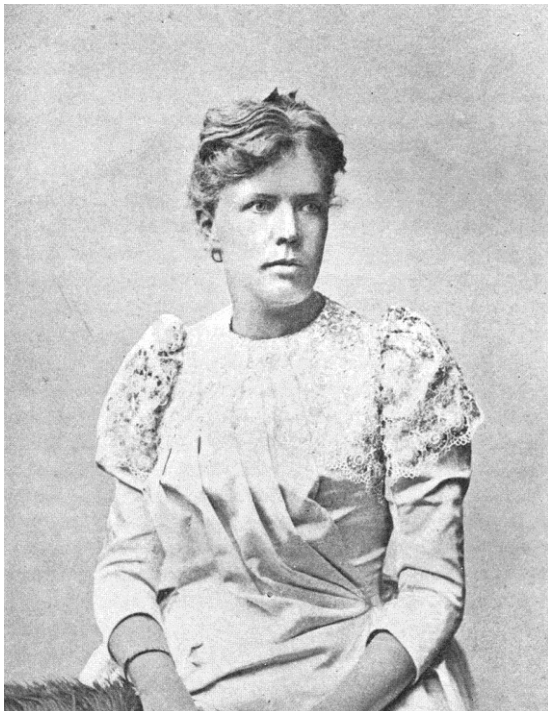
$g$  l'intensité de la force de gravité;

<sup>1</sup> Ce mémoire est le résumé d'un travail auquel l'Académie des Sciences de Paris, dans sa séance solennelle du 24 décembre 1888, a décerné le prix Bordin élevé de 3000 à 5000 francs.

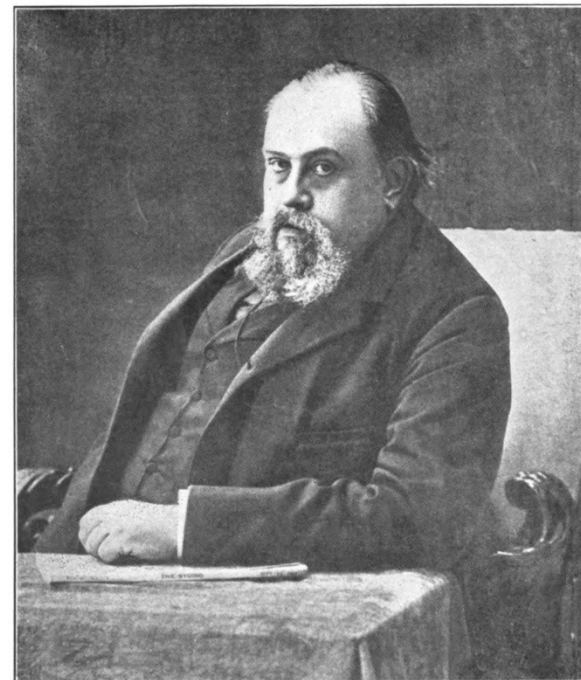
This paper was originally published in *Acta Mathematica* 12 (1889), 177-232. Reprinted with permission.

## Life and death.

- 1887: death of Aniouta – Anne Jaclard.

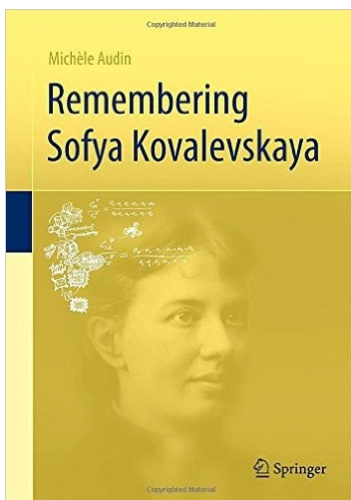
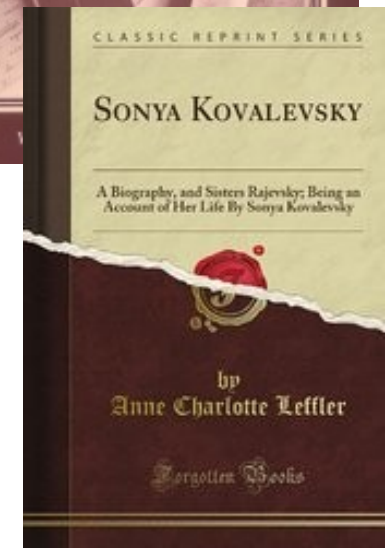
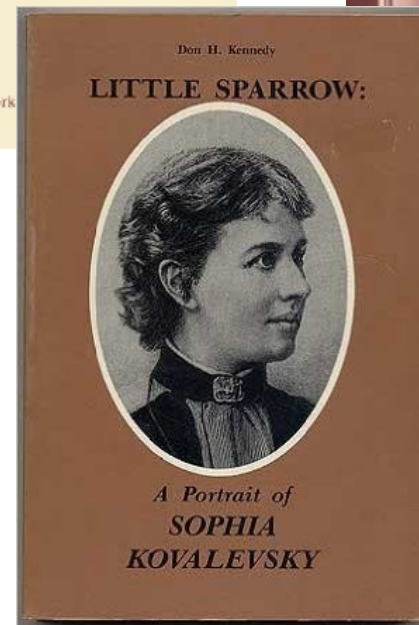
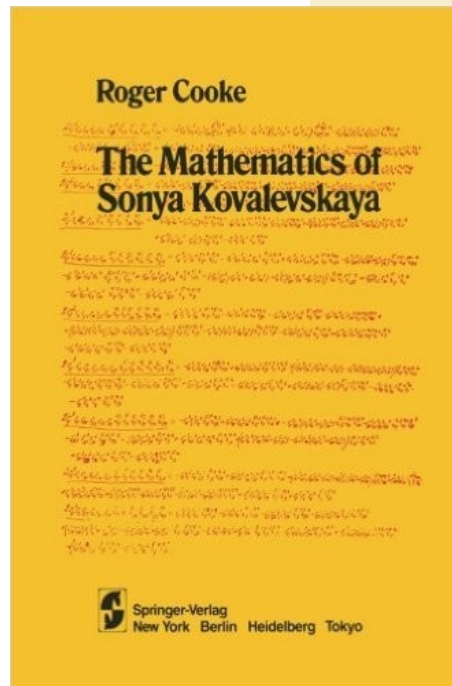
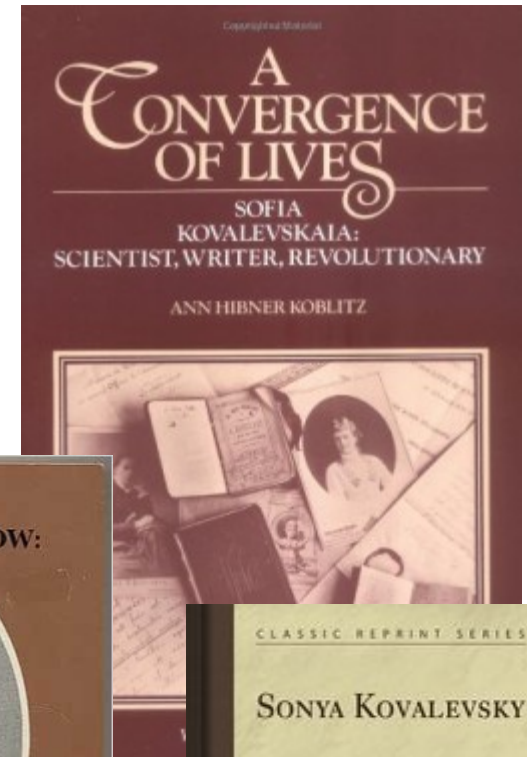
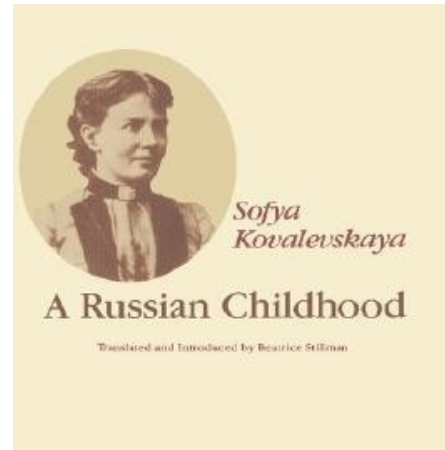
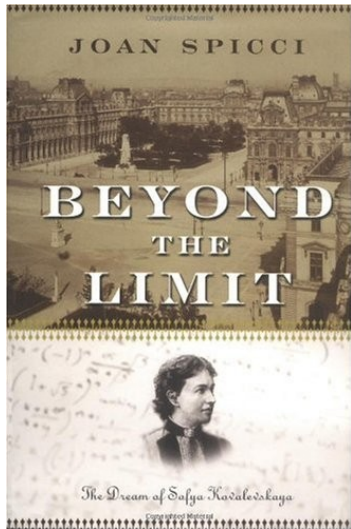


Anne-Charlotte Leffler



Maksim Kovalevsky

From then till now.

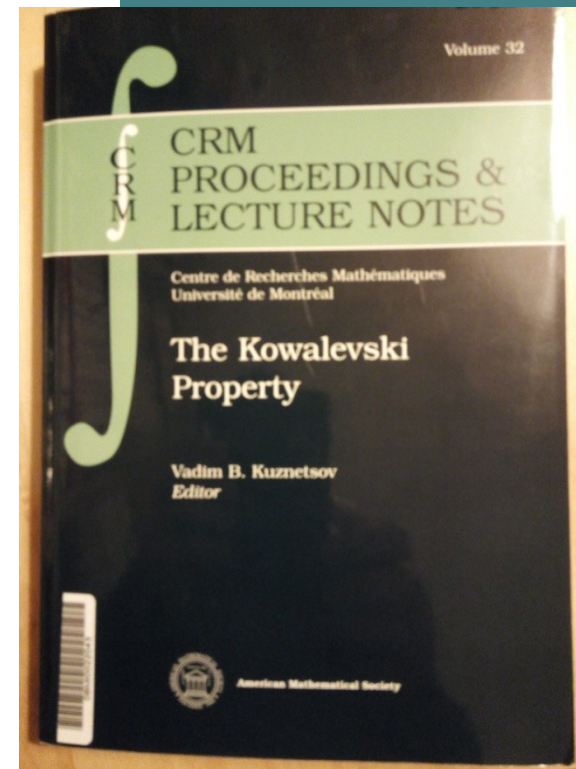


The Sofia Kovalevskaya Award.

# From then till now.

## List of Participants Kowalevski Workshop (MMRD) University of Leeds April 12–15, 2000

- |   |                                       |
|---|---------------------------------------|
| S. Abenda (Bologna)                       | N. Kitanine (St. Petersburg and York) |
| E. D. Bekukotos (Kiev)                    | I. V. Komarov (St. Petersburg)        |
| Yu. Beresit (New York)                    | Y. Koemann-Schwarzbach (Paris)        |
| A. Bobenko (Berlin)                       | V. Kuznetsov (Leeds)                  |
| H. Braden (Edinburgh)                     | D. Lebedev (Moscow)                   |
| R. Caserio (Coimbra)                      | S. Lombardo (Leeds and Roussé)        |
| O. Chalykh (Loughborough)                 | F. Magri (Milano)                     |
| E. Corrigan (York)                        | J.-M. Maillet (Lyon)                  |
| S. Derkachov (St. Petersburg and Leipzig) | D. Markushevich (Lille)               |
| H. Dullin (Loughborough)                  | I. Marshall (Lausanne)                |
| J. C. Eilbeck (Heriot-Watt)               | V. Matvosev (Chelyabinsk and Warwick) |
| V. Enufskil (Kiev and Heriot-Watt)        | M. Mazzocco (Oxford)                  |
| B. Enriquez (Paris)                       | O. McCarthy (Ball)                    |
| G. Falqui (Trieste)                       | A. Mikhailov (Leeds)                  |
| Yu. Fedorov (Moscow)                      | P. van Moerbeke (Louvain-la-Neuve)    |
| M. Feigin (Loughborough)                  | F. W. Nijhoff (Leeds)                 |
| E. Ferapontov (Loughborough)              | D. F. Parker (Edinburgh)              |
| A. Fordy (Leeds)                          | H. Pfeiffer (Cambridge)               |
| B. Goffet (Saclay)                        | L. A. Plovan (Baha Blanca)            |
| A. Grunbaum (Berkeley)                    | E. Previato (Boston)                  |
| L. Haine (Louvain-la-Neuve)               | S. Puttock (Leeds)                    |
| R. Heredero (Madrid)                      | S. Rauch-Wojciechowski (Linköping)    |
| K. Hikami (Tokyo)                         | N. Roshetikhin (Berkeley)             |
| A. Hone (Adelaide)                        | P. Richter (Bremen)                   |
| J. Hurtubise (Montreal)                   | S. Saito (Tokyo)                      |
| A. Its (Indianapolis)                     | M. Salerno (Salerno)                  |
| A. Kapaev (St. Petersburg)                | M. Semenov-Tian-Shansky (Dijon)       |
| A. Kitaev (St. Petersburg and Adelaide)   | E. Sklyanin (St. Petersburg)          |



## LIST OF PARTICIPANTS

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- |                           |
|---------------------------|
| V. Sokolov (Moscow)       |
| Yu. Suris (Berlin)        |
| T. Takebe (Tokyo)         |
| V. Terras (Lyon)          |
| P. Vanhovecke (Poitiers)  |
| C. Verhoeven (Brussels)   |
| A. Veselov (Loughborough) |
| F. Wagner (Cambridge)     |
| K. Wakatsuki (Tokyo)      |
| A. Walker (Leeds)         |
| M. Yan (Hong Kong)        |
| M. Yoshida (Kyushu)       |



« It is impossible to be a mathematician without being a poet in soul. »