Fachbereich Mathematik und Statistik
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REAL ALGEBRAIC GEOMETRY II

Exercise Sheet 10

Convex valuations and the Baer-Krull theorem

Exercise 32
(6 points)

Let $G = (\mathbb{Q}^{>0}, \cdot, <)$ be the ordered multiplicative group of positive rationnal numbers. What is the cardinality of the set of orderings on $\mathbb{Q}((G))$ that are $v_{\min}$-compatible?

Exercise 33
(4 points)

Let $(K, \leq)$ be an ordered field with natural valuation v. Show that there is an ordered field extension L/K with exactly two v-compatible orderings for the natural valuation w on L.

Exercise 34
(4 points)

Let $(L, w)/(K, v)$ be an immediate extension of valued fields. Show that any v-compatible ordering on K extends into a w-compatible ordering on L.

Exercise 35 (bonus)
(6 points)

Let $(\Gamma, \leq)$ be a linearly ordered set. Show that there is an ordered field whose principal rank is the oder type of $(\Gamma, \leq)$ and which embeds into any ordered field with that principal rank.

*Please hand in your solutions by **Thursday, 6 July 2023, 10:00** (postbox 14 in F4).*