

## REAL ALGEBRAIC GEOMETRY II

### Exercise Sheet 11

#### Convex valuations and exponential fields

The references in Exercises 37 and 38 are to Skript 21, SoSe 2019.

#### **Exercise 36**

(4 points)

Let  $(K, \leq)$  be an ordered field which is root closed for positive elements. Let  $e$  be an exponential on  $K$ . Moreover, let  $a \in K$  such that  $e(a) = 2$ . Show that the map

$$f: K \longrightarrow K; x \mapsto e(ax)$$

defines a  $v$ -compatible exponential on  $K$  where  $v$  is the natural valuation.

#### **Exercise 37**

(4 points)

Prove Hilfslemma 2.4.

#### **Exercise 38**

(4 points)

Using Hilfslemma 2.4 and Proposition 2.6, prove the theorem of additive lexicographic decomposition (Theorem 2.3).

*Please hand in your solutions by **Thursday, 13 July 2023, 10:00** (postbox 14 in F4).*