Fachbereich Mathematik und Statistik
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Real Algebraic Geometry II

Exercise Sheet 7

Fields of generalised power series

Exercise 25

(4 points)

Let k be an Archimedean field and let G be an ordered abelian group. Set $\mathbb{K} := k((G))$.

a) Find an order-preserving isomorphism of groups from $v(\mathbb{K}^\times)$ to G.

b) Show that there is a unique order-preserving isomorphism between the Archimedean ordered fields $\overline{\mathbb{K}}$ and k.

Exercise 26

(4 points)

Let k be an Archimedean field which is square root closed for positive elements, i.e. for any $a \in k^{>0}$, there exists a $b \in k$ with $b^2 = a$. Let G be an ordered abelian group which is 2-divisible, i.e. for any $g \in G$, there exists an $h \in G$ such that $h + h = g$. Let $\mathbb{K} = k((G))$.

Let $\varepsilon \in \mathbb{K}$ with $\mathrm{support}(\varepsilon) \subseteq G^{>0}$ and let $\alpha \in \mathbb{Q}^{>0}$. We have an element

$$\sum_{n=0}^{\infty} \frac{(\alpha)_n}{n!} \varepsilon^n \in \mathbb{K},$$

where

$$(\alpha)_n = \prod_{k=0}^{n-1} (\alpha - k).$$

a) Show that

$$\left(\sum_{n=0}^{\infty} \frac{(\frac{1}{2})_n}{n!} \varepsilon^n\right)^2 = 1 + \varepsilon.$$

b) Deduce that $\mathbb{K}$ is square root closed for positive elements.

Please hand in your solutions by ***Thursday, 15 June 2023, 10:00*** *(postbox 14 in F4).*