Fachbereich Mathematik und Statistik
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Real Algebraic Geometry II

Exercise Sheet 8

Fields of generalised power series II

Exercise 25

(4 points)

Let k be an Archimedean ordered field and let G be an ordered abelian group. Let $\mathbb{K} = k((G))$. Moreover, let $\tilde{\mathbb{K}} \supseteq \mathbb{K}$ be an algebraic closure of $\mathbb{K}$ and let i be an element in $\tilde{\mathbb{K}}$ with $i^2 = -1$.

Show that

$$\mathbb{K}(i) \cong k(i)((G)).$$

Exercise 26

(4 points)

Let k be a non-Archimedean ordered field. Let w be a valuation on K with valuation ring K_w and valuation ideal I_w. In order to complete the proof of Proposition 3.2, Skript 17 of the RAG II 2019 lecture, show that 5) $\Longrightarrow$6), 6) $\Longrightarrow$ 7) and 7) $\Longrightarrow$ 1.

Exercise 27

(4 points)

a) Show that $|\mathbb{Q}^{\mathrm{rc}}((\mathbb{Q}))| = 2^{\aleph_0}$.

(*Hint: use without proof that* $\aleph_0^{\aleph_0} = 2^{\aleph_0}$.)

b) Find a countable, non-Archimedean real closed subfield of $\mathbb{Q}^{\mathrm{rc}}((\mathbb{Q}))$.

*Please hand in your solutions by **Thursday, 22 June 2023, 10:00** (postbox 14 in F4).*